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Miss Distances in Tactical Missile Intercepts

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OUTLINE

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by

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ABSTRACT

Endoatmospheric tactical missiles are hard to hit because their response to their environment may not be too different from optimal escape maneuvers. For varying or random accelerations, optimal guidance could increase errors. Maneuvering missiles, weapons, and debris are also hard to hit. Breaking up could cause erratic motions of the target in about the same frequency range as optimal evasive maneuvers.

I. INTRODUCTION

Historically, it has been difficult for interceptors to hit tactical endoatmospheric missiles. This note discusses why and gives some estimates of miss distances for accelerating targets, for which there appear to be fundamental limits.

II. HIGH-ALTITUDE CRUISE MISSILES

It has been hard to hit high-altitude supersonic cruise missiles because of their barometric porpoising on density waves in the atmosphere. This section discusses why.

One reason is given in <u>Tactical and Strategic Missile Guidance</u>,[1] which gives the normalized miss distance as a function of the flight time t_F normalized by the guidance time T for an interceptor with a typical 5th order guidance and guidance ratio $N' = 5$ against a missile executing optimal evasion maneuvers of maximum acceleration n_T g's. It shows the miss distance in feet to be about $125\ T^2 n_T$. At high altitudes, T might be around 1 s, so a porpoising maneuver with $n_T \approx 1$ g would give a miss distance $\approx 125\ (1\ \text{s})^2\ 1\ \text{g} \approx 125$ ft, which certainly qualifies as a miss.

It is interesting that the optimal escape maneuver, which consists of the missile alternately pulling $+n_T$ and $-n_T$ g's for a few seconds, is not too far from what a high-altitude supersonic cruise missile might do by accident due to irregular inputs from its barometric guidance.

III. LOW-ALTITUDE CRUISE MISSILES

A low-altitude subsonic cruise missile randomly pulling ≈ 1 g turns during ingress could produce much the same effect on an interceptor. Presumably, the interceptors would have somewhat better performance at low altitudes, but against an interceptor with $T = 0.5$ s, the miss distance would be $\approx 125\ (0.5\ \text{s})^2\ 1\ \text{g} \approx 30$ ft, which is far from the few feet needed for hit to kill.

Moreover, at low altitudes, noise and radome parasitics may make it necessary to use lower navigation ratios and slower response times than those assumed above, which would increase the miss distance.[2]

The usual answer would be to use augmented proportional navigation or optimal predictors to make up for acceleration. But developed augmented and optimal guidance work for constant acceleration. For the varying or random accelerations treated here, they could increase the error.

IV. THEATER MISSILES

The lower tier of ground-based, kinetic-energy interceptors (GBIs) have accelerations, velocities, and guidances adequate for intercepting predictably decelerating terminal objects.[3] They could address most of the threats posed in the papers[4,5] on which the current global protection against limited strikes (GPALS) is based.[6,7]

Maneuvering missiles, weapons, and debris are harder to hit.[8,9] Patriot may have done about as well as is possible in its altitude range,[10,11] which is about where deceleration is a maximum. Adding asymmetrical bodies--accidental or otherwise--causes transverse accelerations, which are even harder to contend with. In the Gulf War, the debris and upper stages hanging off SCUDs caused them to perform exotic helices and jinking that sometimes threw off the even Patriot's robust navigation.

The problem is discussed in the next section. The analysis was initially performed in support of the air defense initiative (ADI) concepts that sought to do pure hit to kill, which made it necessary to look back into the guidance and error budgets for endoatmospheric interceptors. In the process, it investigated the errors in intercepting supersonic cruise missiles, which proved to be common to theater missiles.[12]

Breaking up, alone, could cause erratic motions of the target in about the same frequency range as that for the optimal evasive maneuver. Those motions could foil predictive navigation and generate miss distances of meters in radar-driven, slow-reacting interceptors. If so, SDIO could finish the next decade of FSEDs with a "hit-to-kill" interceptor with an on-board radar that would still need 50-100 kg of HE on board to kill.

Thus, it wouldn't be bad to have space-based interceptors or directed energy weapons (DEWs) as backups that could kill in boost or midcourse before this accidental jinking started. In the boost phase, DEWs can handle very short-range launches much better than SBIs, so they are preferred for the current threat. They can also discriminate the accidental and intentional junk, if necessary, as discussed in recent exchanges.[13,14]

Thus, theater missile defenses are in a limited design space.[15] For ERINT-altitude intercepts they are stuck with radars, radome-slope errors, and, hence, long response constants, which are fundamental and may not be engineered around affordably. Arrow or THAAD with purely IR guidance could do better against unintentional jinking, but probably not for intentional jinking.

V. ANALYSIS

The source of current concepts' problem with acceleration is straightforward. The linearized proportional guidance equation is

$$y'' = a_T - n_C, \tag{1}$$

where the primes denote differentiation with respect to time and the target acceleration is taken to be $a_T = A \sin wt$ with A and w parameters. The proportional navigation missile acceleration is

$$n_C = N'Vy'/R \approx N'Vy'/Vt_F = N'y'/t_F \equiv ky', \tag{2}$$

where transverse distances are taken to be small, $N' \approx 3$ to 5 is the dimensionless effective navigation ratio (guidance gain), V is the closing velocity, and R is the average range. Thus,

$$y'' = A \sin wt - ky', \tag{3}$$

whose solution is

$$y = A[(k/w)(1-\cos wt)-\sin wt+(w/k)(1-e^{-kt})]/(w^2+k^2) \tag{4}$$

which is shown in Fig. 1 for w = 1/s and A = 1 g. The top curve is for k = 1; the middle for k = 3; and the bottom for k = 10/s. The curves have peak miss distances of ≈ 17, 7, and 2 m respectively. Stronger guidance reduces the miss distances, but the sinusoidal oscillations do not damp out. Equation (4) shows that for w large the envelope of the error is $y \approx A/w^2$, which is the amplitude of the missile oscillation. For k large, $y \approx A/wk$, which reduces the amplitude of the displacements by a factor w/k.

It might be possible to perform better against a fixed acceleration A, but the accelerations are random. The RMS value of the miss $\surd\langle y^2\rangle$ can also be seen from Eq. (4) to be proportional to $\surd\langle A^2)$, the RMS value of the acceleration. The average miss distances can also be read from Eq. (4). Because of this A dependence, adjusting the guidance to the wrong acceleration can increase errors.

Figure 2 shows the miss distance at $t = t_F = 9$ s, a final time that is neither at the minimum nor the maximum of the displacements, as a function of k for w = 0.5, 1, and 2/s.

For k small, the miss distances are tens of meters. For $k = 5/s$ they are ≈ 4, 3, and 1 m for $w = 0.5$, 1, and 2/s, respectively.

High-gain guidance could reduce the impact of time-varying accelerations to usable levels. Note, however, that $k = 5/s$ corresponds to $N'V/R = 5/s$, or $N' = 5R/V \approx 5/s\ 5\ km \div 1\ km/s \approx 25$, which is extremely large relative to the $N' \approx 3$-5 nominally used. Thus, $N' \approx 5$ gives $k \approx 5\ 1\ km/s \div 5\ km \approx 1/s$, which gives misses of ≈ 5-10 m even for $w = 1$-2/s. Since from Eq. (2), $k = N'/t_F$, where t_F is a random variable that depends on the point in its trajectory in which a target was detected, it is useful to rewrite Eq. (4) in terms of t_F as

$$y = A[(N'/wt_F)(1-\cos wt_F)-\sin wt_F + (wt_F/N')(1-e^{-N'})]t_F^2/[(wt_F)^2+N'^2], \quad (5)$$

which can be averaged over t_F to give the expected miss distance. The general result is complicated but the limits are straight forward. For $wt_F >> N'$, the first two terms in the first bracket are small compared to the third, which averages to

$$\langle y^2\rangle \approx \langle A^2\rangle\langle (wt_F/N')^2\rangle/w^2 \approx (w^2\langle t_F^2\rangle/N'^2)\langle A^2\rangle/w^2, \quad (6)$$

so that $\sqrt{\langle y^2\rangle} \approx w\sqrt{\langle t_F^2\rangle}/N'$ x RMS target oscillation, and the average miss distance is larger than the target oscillation. Thus, random or deterministic high-frequency oscillations are stressing to the interceptor. For $wt_F << N'$, the first term in the first bracket is dominant and gives

$$y \approx A(t_F^2/N'^2)[(N'/wt_F)(w^2t_F^2/2)] \approx (At_F^2/2)wt_F/N', \quad (7)$$

so that the average miss distance is the average target motion times wt_F/N', which is small. In this low-frequency limit, successful intercepts should be possible.

As noted above, at low altitudes, noise and radome parasitics may make it necessary to use lower navigation ratios and slower response times than those assumed. This model can be extended to use the instantaneous acceleration for augmented proportional navigation. The results can be worse.[16] The model calculations discussed here do not include guidance delays, which would make the errors larger.

VI. SUMMARY AND CONCLUSIONS

This note discusses why it is hard to hit tactical endoatmospheric missiles and gives some estimates for typical miss distances. High-altitude supersonic cruise missiles are hard to hit because their barometric porpoising on density waves in the atmosphere can simulate optimal evasive maneuvers. Their optimal escape maneuver, which consists of alternately pulling its maximum positive and negative accelerations for a few seconds, is not too far from what the high-altitude supersonic cruise missile might do by random operation of its barometric guidance.

A low-altitude subsonic cruise missile randomly pulling 1 g turns during ingress could produce much the same effect on an interceptor. The usual answer is to use augmented proportional navigation or optimal predictors to make up for acceleration, but developed augmented and optimal guidance only work for constant acceleration. For varying or random accelerations, they could increase the error.

Maneuvering missiles, weapons, and debris are also hard to hit. Patriot may have done about as well as is possible in its altitude range. Asymmetrical bodies--accidental or otherwise--cause transverse accelerations, which are hard to accommodate. Just breaking up could cause erratic motions of the target in about the same frequency range as that for the optimal evasive maneuver. Those motions could foil predictive navigation and generate miss distances of meters in radar-driven, slow reacting interceptors.

The source of the problem is clear from the linearized proportional guidance equation. The sinusoidal oscillations in error are large and do not damp out. It is clear that hit-to-kill could be difficult against even straight-and-level, low-altitude trajectories. From the calculations and comments above, it would appear that they could be quite difficult against endoatmospheric targets that accelerate by design or accident.

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Fig. 1 Displacement versus time

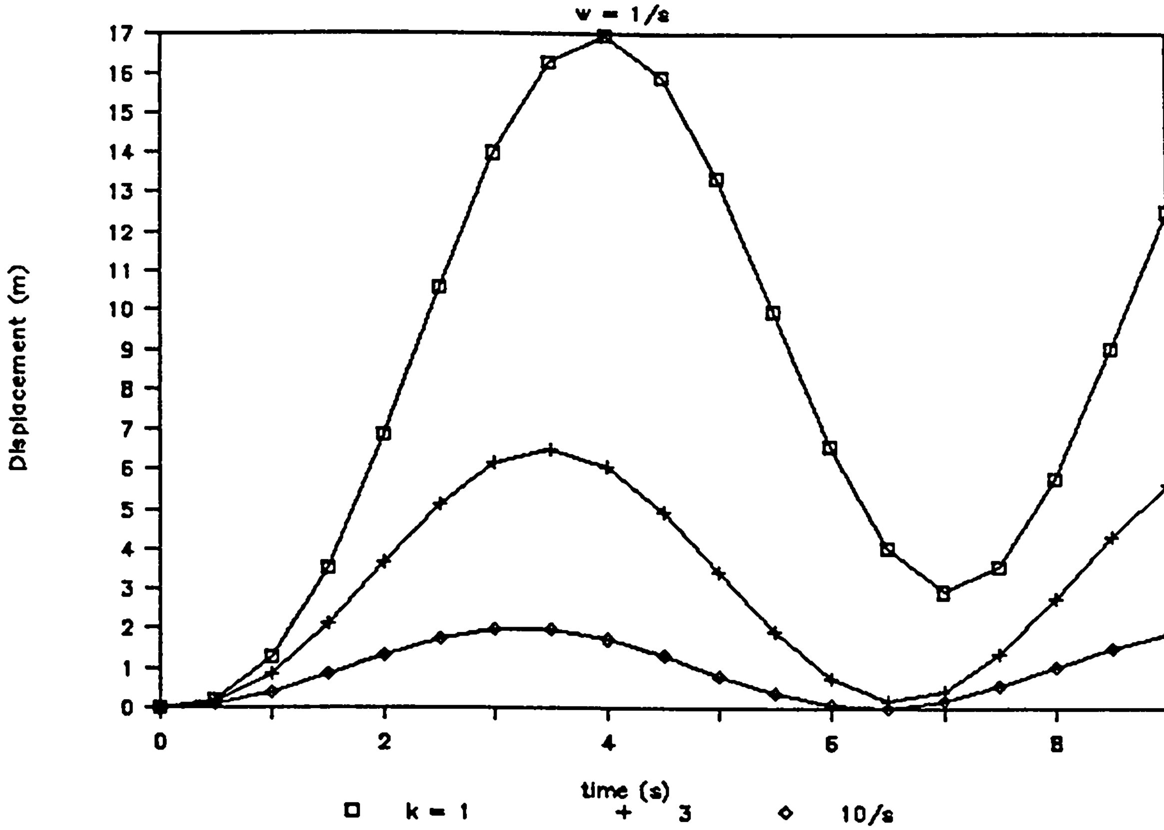

Fig. 2 Displacement versus rates